Statistics

Lecture 10

Feb 19-8:47 AM

Prob. of making a correct guess is given to be .1.

$p = .1 \qquad q = .9 \qquad \mu = \frac{1}{p} = \frac{1}{.1} = 10$

$\sigma^2 = \frac{q}{p^2} = \frac{.9}{.1^2} = 90 \qquad \sigma = \sqrt{\sigma^2} = \sqrt{90} \approx 9.5$

P(First correct guess happens before 5th trial)

$P(x < 5) = P(x \le 4) = \text{geometcdf}(.1, 4) = \boxed{.3439}$

P(first correct guess happens after 3rd trial)

$P(x > 3) = P(x \ge 4) = 1 - P(x \le 3)$

we don't want 3 | 4 we want

$= 1 - \text{geometcdf}(.1, 3) = \boxed{.729}$

Apr 16-6:51 PM

The average number of spam calls during any given day is 10.

P(You get 12 spam calls in any given day)

$P(x=12) = \text{Poissonpdf}(10, 12) = \boxed{.095}$

P(You get at least 8 spam calls in any given day)

$P(x \geq 8) = 1 - P(x \leq 7) = 1 - \text{Poissoncdf}(10, 7)$

we don't want 7 | 8 we want

$= \boxed{.780}$

P(You get 5 or 20 spam calls in any given day)

$= P(x=5 \text{ or } x=20) = P(x=5) + P(x=20)$

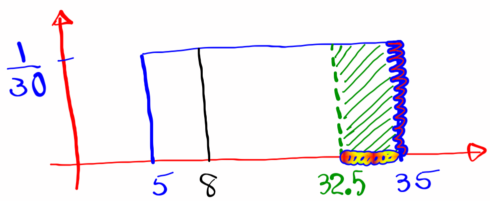
$= \text{Poissonpdf}(10,5) + \text{Poissonpdf}(10,20)$

$= \boxed{.040}$

SG 17

Apr 16-6:58 PM

Consider a uniform Prob. dist. for all values from 5 to 35.



1) $P(x=8) = 0$

2) $P(x>32.5)$

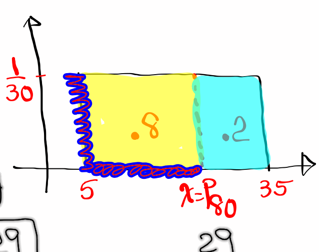
$= (35-32.5)\cdot\frac{1}{30}$

$= \frac{2.5}{30} = \boxed{\frac{1}{12}}$

3) Find P_{80}

80% below — Left Area .8

20% above — Right Area .2



$(x-5)\cdot\frac{1}{30} = .8$

$x-5 = 30(.8)$

$x-5=24$

$\boxed{x=29}$

Apr 16-7:06 PM

Find $P(-1.2 < Z < 1.8)$

Standard Normal Prob. dist.

symmetric, bell-shape, Total Area = 1

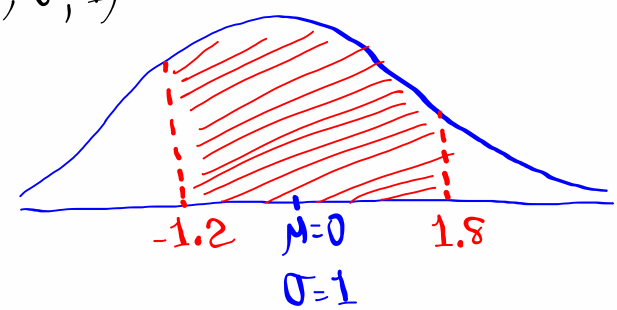
$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$

L U μ σ $\mu = 0$, $\sigma = 1$

normalcdf(-1.2, 1.8, 0, 1)

(−)

$= \boxed{.849}$



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$P(Z < 2.326)$

shade left

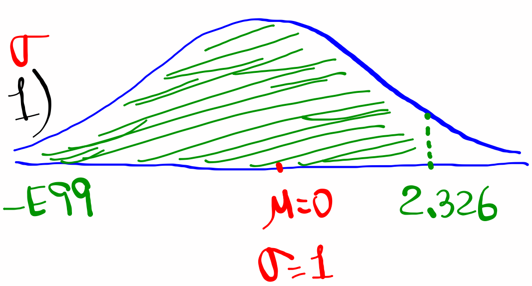
L U μ σ

= normalcdf(-E99, 2.326, 0, 1)

(−)

$\boxed{\text{2nd}}$ $\boxed{,}$

$= \boxed{.990}$

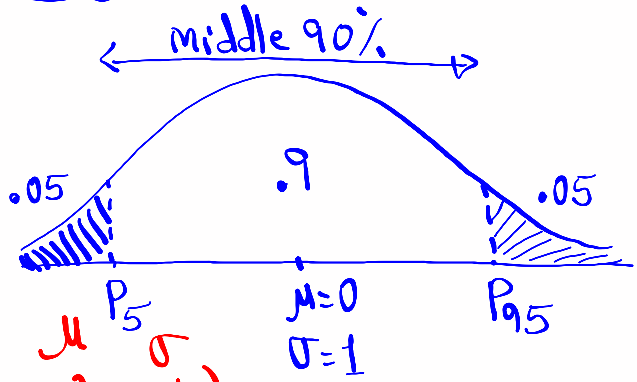


Apr 16-7:18 PM

Find two Z-Values, Rounded to 3-decimal places, that separate the middle 90% from the rest.

$1 - .9 = .1$

$.1 \div 2 = .05$



Left Area

μ σ

$P_5 = \text{invNorm}(.05, 0, 1)$

$= \boxed{-1.645}$

$P_{95} = \text{invNorm}(.95, 0, 1)$

$= \boxed{1.645}$

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Normal Prob. dist.:

1) Use x, $P(x = c) = 0$

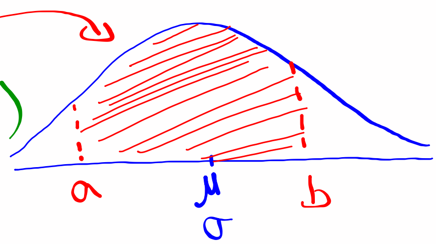
2) Symmetric, bell-Shape, Total area $= 1$

3) Mean = Mode = Median

4) μ & σ are given in the problem.

$P(a < x < b)$

$= \text{normalcdf}(L, U, \mu, \sigma)$



$N(\mu, \sigma)$

Normal

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Given $N(75, 10)$

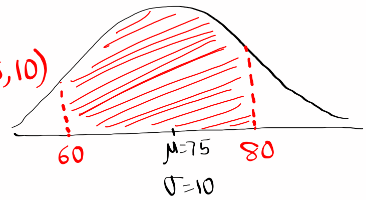
$\mu = 75$ $\quad \sigma = 10$

Normal Prob. dist.

$P(60 < x < 80)$

$= \text{normalcdf}(60, 80, 75, 10)$

$= \boxed{.625}$

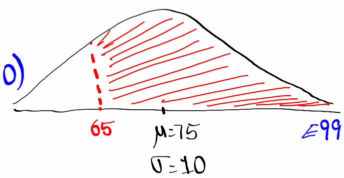


$P(x > 65)$

shade right

$= \text{normalcdf}(65, E99, 75, 10)$

$= \boxed{.841}$ $\quad \boxed{2nd}\boxed{,}$

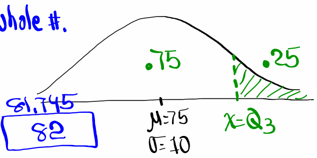


Find $x = Q_3$, Round to whole #.

75% below $\quad$ 25% above

81.745

$x = \text{invNorm}(.75, 75, 10) \approx \boxed{82}$



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Ages of students are normally dist. with the mean of 32 and standard dev. of 5.

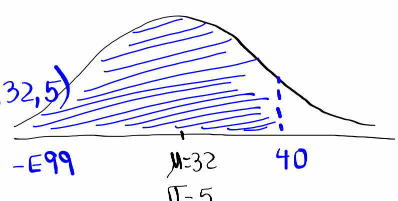
$N(32, 5)$

Find the prob. that one randomly selected student is below 40 yrs old.

$P(x < 40)$

$= \text{normalcdf}(-E99, 40, 32, 5)$

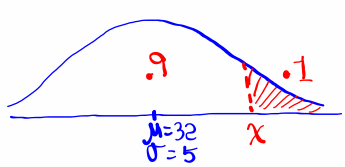
$= \boxed{.945}$



Find the age that separates the $\boxed{\text{top } 10\%}$ from the rest. Round $\boxed{\text{to whole \#}}$

$x = P_{90} = \text{invNorm}(.9, 32, 5)$

$= 38.408 \approx \boxed{38}$



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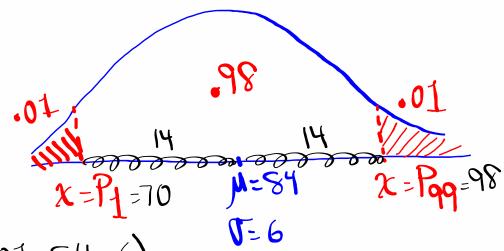
Exam scores are normally dist. with $\mu = 84$ and $\sigma = 6$.

Find two scores, round to whole numbers, that separate the $\boxed{\text{middle } 98\%}$ from the rest.

$1 - .98 = .02$

$.02 \div 2 = .01$



$x = P_1 = \text{invNorm}(.01, 84, 6)$

$= 70.042 \approx \boxed{70}$

$x = P_{99} = \text{invNorm}(.99, 84, 6) = \boxed{98}$

$\boxed{\text{SG } 18 \checkmark \; \& \; \text{SG } 19 \checkmark}$

Apr 16-7:49 PM

Clear all lists

Reset all lists

Consider the following Population

2, 4, 6, 8

Store it in L1

Use $\boxed{\text{1-Var Stats}}$ with L1 → to find

$\mu = \bar{x} = \boxed{5}$

$\sigma = \sigma_x = \boxed{2.236}$

$\sigma^2 = \boxed{5}$ Frac.

Let's take all sample of Size 2 $\boxed{\text{with replacement}}$ from this population

2,2	2,4	2,6	2,8
4,2	4,4	4,6	4,8
6,2	6,4	6,6	6,8
8,2	8,4	8,6	8,8

16 Samples of Size 2

Now find $\bar{x}$ of each Sample.

2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8

16 means

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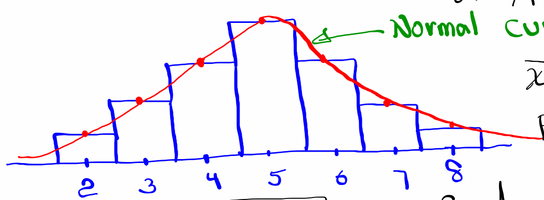
2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8

16 means

$\bar{x}$	$P(\bar{x})$
2	1/16
3	2/16
4	3/16
5	4/16
6	3/16
7	2/16
8	1/16

Draw Prob. dist. Histogram



$\bar{x} \to L2$

$P(\bar{x}) \to L3$

Use $\boxed{\text{1-Var Stats}}$ with L2 and L3 → find

$\mu = \bar{x} = 5$

$\sigma = \sigma_x = 1.581$

$\sigma^2 = \frac{5}{2}$ frac

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Clear all lists.

Store the following Population in L1.

1 3 5 7 9

Use $\boxed{\text{1-Var Stats}}$ with L1 → to find

$\mu = 5$

$\sigma = 2.828$

$\sigma^2 = 8$

Now find all possible Samples of Size 2 with replacement from this population.

1,1	1,3	1,5	1,7	1,9
3,1	3,3	3,5	3,7	3,9
5,1	5,3	5,5	5,7	5,9
7,1	7,3	7,5	7,7	7,9
9,1	9,3	9,5	9,7	9,9

25 Samples with Size 2

Now find $\bar{x}$ for each Sample

1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9

25 means

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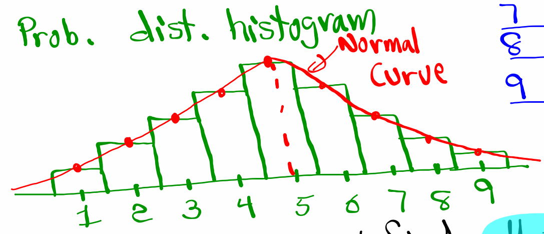
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9

25 means

$\bar{x}$	$P(\bar{x})$
1	1/25
2	2/25
3	3/25
4	4/25
5	5/25
6	4/25
7	3/25
8	2/25
9	1/25

Prob. dist. histogram



$\bar{x} \to L2$

$P(\bar{x}) \to L3$

Use $\boxed{\text{1-Var Stats}}$ with L2 & L3 → Find

$\mu_{\bar{x}} = 5$

$\sigma_{\bar{x}} = 2$

$\sigma^2_{\bar{x}} = 4 = \frac{8}{2}$

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Central Limit Theorem

$$\boxed{\mu_{\bar{x}} = \mu}$$

$$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}$$

$$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$$

Suppose $\mu = 2500$, $\sigma = 40$ and we take Samples of Size 25

$$\mu_{\bar{x}} = \mu = 2500$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{40}{\sqrt{25}} = \frac{40}{5} = 8$$

Apr 16-8:44 PM

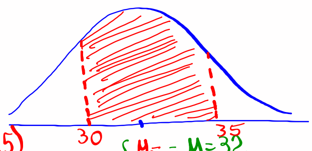
Ages of all students are normally dist. with $\mu = 32$ and $\sigma = 5$. $N(32, 5)$

If we randomly select $\overset{n=4}{\boxed{4 \text{ students}}}$, find the prob. that $\overset{\bar{x}}{\boxed{\text{their mean}}}$ age will be $\boxed{\text{between 30 and 35}}$

$P(30 < \bar{x} < 35)$

$= \text{normalcdf}(30, 35, 32, 2.5)$

$= \boxed{.673} \approx 67.3\%$

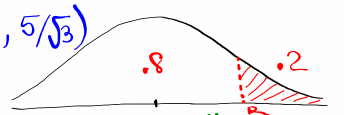


CLT $\begin{cases} \mu_{\bar{x}} = \mu = 32 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{5}{\sqrt{4}} = 2.5 \end{cases}$

Find $\bar{x} = P_{80}$ for randomly selected groups of 3 students. Round to 1-decimal.

$\bar{x} = \text{invNorm}(.8, 32, 5/\sqrt{3})$

$\approx \boxed{34.4}$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 32 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{5}{\sqrt{3}} \end{cases}$

Apr 16-8:48 PM

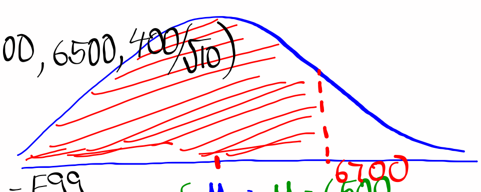
Salaries of nurses are normally dist. with the mean of \$6500/mo. with standard deviation of \$400/mo. $N(6500, 400)$

If we randomly select $\overset{n=10}{\boxed{10 \text{ nurses}}}$, find the Prob. that $\overset{\bar{x}}{\boxed{\text{their mean salary}}}$ is below \$6700/mo.

$P(\bar{x} < 6700)$

$= \text{normalcdf}(-E99, 6700, 6500, 400/\sqrt{10})$

$= \boxed{.943}$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 6500 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{400}{\sqrt{10}} \end{cases}$

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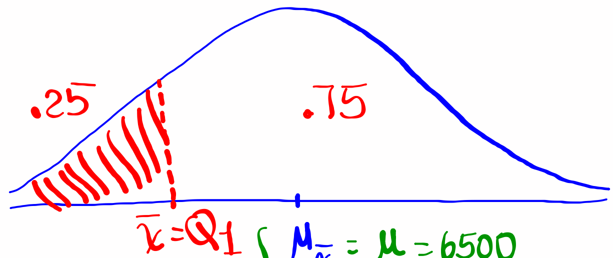
find $\bar{x} = Q_1$ for randomly selected groups of 16 nurses.



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 6500 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{400}{\sqrt{16}} = 100 \end{cases}$

$\bar{x} = \text{invNorm}(.25, 6500, 100)$

$= \boxed{\$6433}$

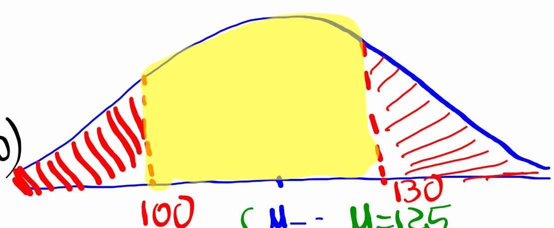
Apr 16-9:07 PM

Given $N(125, 20)$, Sample Size of 4. $\rightarrow n = 4$

$P(\bar{x} < 100 \quad \text{OR} \quad \bar{x} > 130)$

$= 1 - P(100 < \bar{x} < 130)$

$= 1 - \text{normalcdf}(100, 130, 125, 10)$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 125 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{20}{\sqrt{4}} = 10 \end{cases}$

$= \boxed{.315}$

Apr 16-9:11 PM

Given $N(85, 7)$ with Sample Size 4

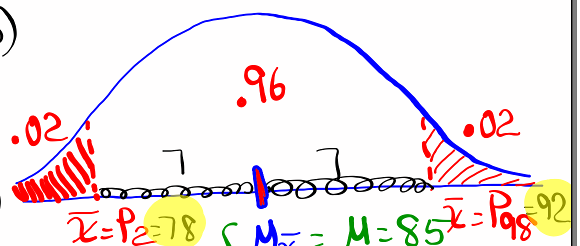
Find two $\bar{x}$ round to whole numbers that separate the middle 96% from the rest.

$\bar{x} = P_2 = \text{invNorm}(.02, 85, 3.5)$

≈ 78 ✓

$\bar{x} = P_{98} = \text{invNorm}(.98, 85, 3.5)$

≈ 92



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 85 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{7}{\sqrt{4}} = 3.5 \end{cases}$

$\boxed{\text{SG 20 ✓} \quad \text{SG 21 ✓}}$